Calculating Labor Costs with an Overlapping Generations Model

For the purpose of this paper, individuals are born in years, groups of individuals born in the same year are "generations," each year of a generation's timeline is a "segment," and all segments that overlap a referenced generation are considered a "structure."

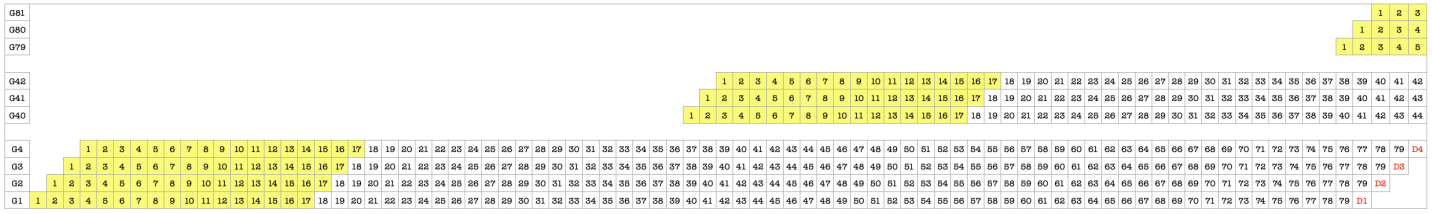
Imagine a new society is charged with implementing a National Defense Strategy (NDS) for the first time in its existence for a period of one year. It would be logical for that society to construct a list of potential NDS factors that would need to include, at a minimum: population, population demographics, geographical area, coastlines, technological advantages, allies, enemies, proximity to enemies, relative religious and political positions, natural and monetary resources, and competing needs for available resources. Once established, the society could determine the number of laborers needed to implement a selected NDS for one year (defined as L_1) and appropriate compensation (defined as C_1). Thus, the Total Labor Costs for year one (TLC_1) are: $L_1 * C_1 = TLC_1$

If individuals lived until their 80th birthday on average, Generation One's (G_1) cost would be $TLC_1 + 1/2\ TLC_2 + 1/3\ TLC_3 + ... + 1/79\ TLC_{79}$ which could be represented by a string of segments each representing a single year of TLC_n ending in D_1 (birthdate death expectancy of G_1).

Generation One's Timeline

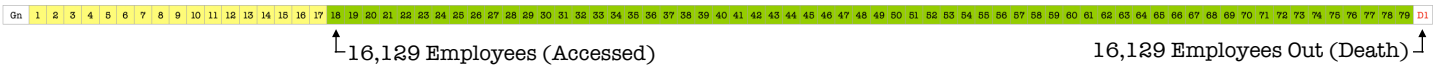
Each year, a new generation (G_n) is born who will inherit a NDS from the pervious set of generations and will share in the structured TLC over their lifetime from year one (Y_n) until their death (D_n). Individuals benefit from the NDS during early years of life and will contribute directly toward future labor costs through taxes during working years or as a laborer for the NDS after reaching an established accession age, assumed to be age 18 or older. Generation Two (G_2) overlaps G_1 by one year, as does every subsequent generation until G_{80} replaces G_1 upon birth and death (D_1) respectively.

Overlapping Generations' Timelines



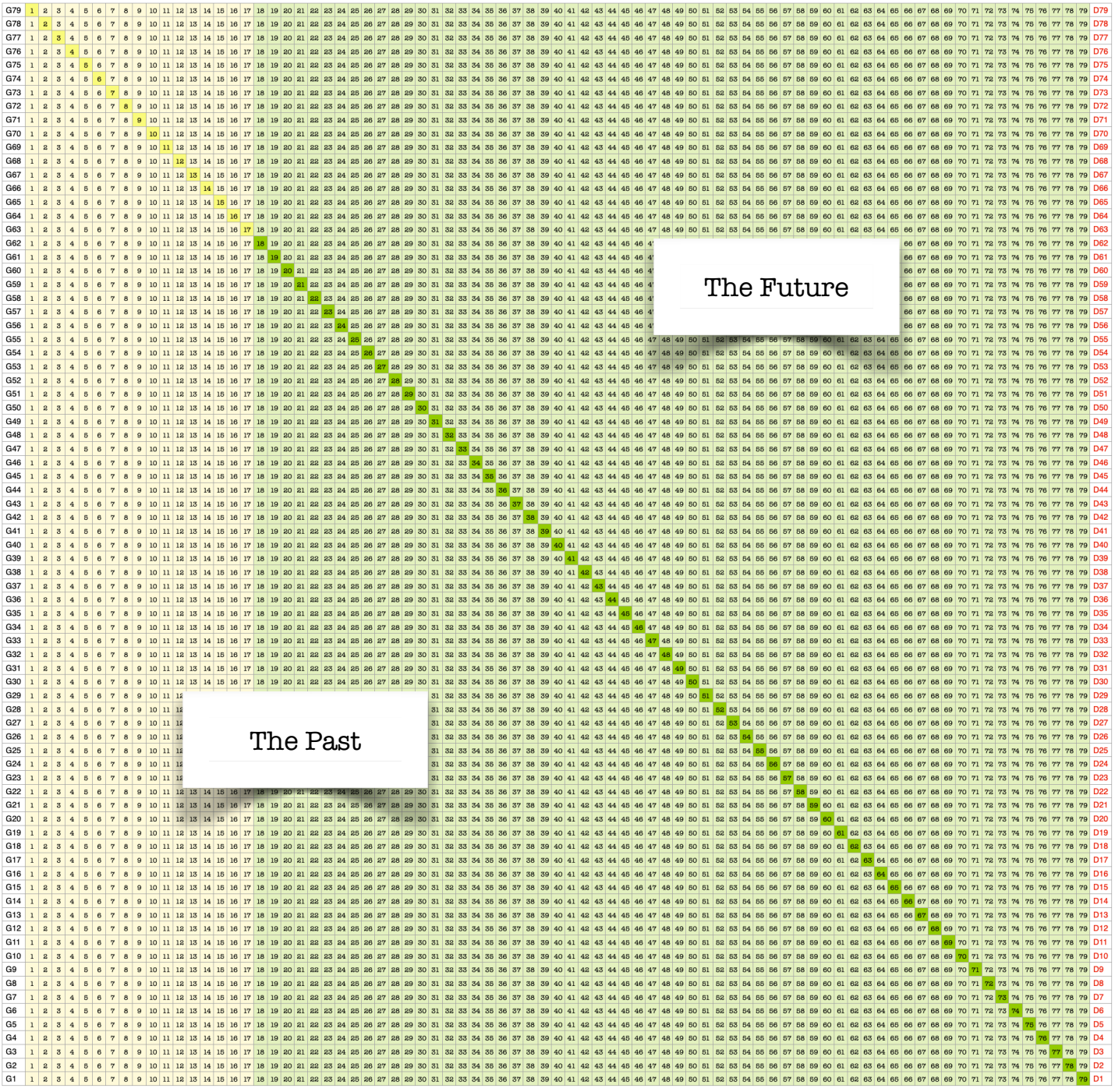
The per generational share of labor supply finds harmony or equilibrium when newly accessed generational laborers are equal to the number of past generational laborer deaths, and each generation supplies an equal share of laborers during the defined workable years [NDS labor requirement divided by (Life Expectancy less Pre-accession Years)]. It is at this point of harmony or equilibrium that TLC per generation in the system can be forecasted and balanced equitably if so desired. Holding all other variables constant, if a new generation had more individuals than a previous generation, the per person cost would be lower for the new generation, or a previous generation could pass a higher share of its TLC to the new generation unnoticed. It took 80 generations for the first generation to be replaced by a new generation. If L is the number of laborers required by the NDS and cannot be filled by individuals below the age of 18, and the average life expectancy is 79.9 years, there would be 62 laboring years and each generation would supply $1/62^{th}$ of the required laborers. If the NDS required 1 million laborers, 62 generations would supply approximately 16,129 laborers over a 62-year period from age 18 to age 79, and 79 generations, age 1 to 79 would share in the benefits and a fractional share of TLC.

Generation Share of National Defense Labor



If we collapse the overlapping generation timelines on top of one another, and highlight the current age of each generation in the 79th year, we would see the ages of the supplied 16,129 laborers from each generation G_1 to G_{62} needed to meet the NDS requirement of 1 million laborers (62 * 16,129 ≈ 1 million), and the ages of the young generations G_{63} to G_{79} that will be needed as laborers in the future. In the 80th year, G_1 would pass and be replaced by G_{80}, but the structure of laborers and relative ages supplied by each generation would remain constant and everything would just "click" forward one year in time. The NDS structure does not change from a total of 1 million individuals and has achieved harmony or equilibrium. The cost to each generation is 1/79th of TLC_{80}.

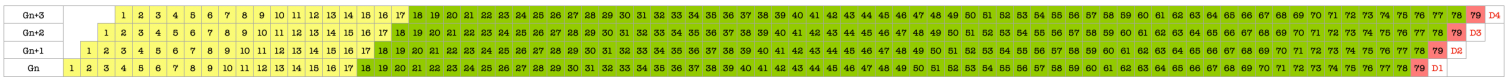
Generation Ages in Year 79



In year 80, let's assume that society reviews its NDS factors list and has a surplus of resources and or laborers and it implements a new rule that allows laborers to retire after 61 years of service in the 79th year of age (one year of retirement benefits). Generation G_2 laborers are now in their 79th year-of-age and immediately receive this new benefit and retire, G_3 and all future generations will receive this benefit in the 79th year of their timeline with G_{80} receiving the retirement benefit in Y_{159}. The NDS structural costs of TLC_{80} have increased with the new element of Retirement Cost (R_{80}) and can be shown as $(L_{80} * C_{80}) + (L_{79} * R_{80}) = TLC_{80} > L_{79} * C_{79} = TLC_{79}$

The retirement costs of R_n are distributed over 79 Generations and are acceptable to society so they are absorbed and go relatively unnoticed. Each generation after G_1 has a higher cost of NDS because it must now pay for new retirement costs associated with a previous generation and supply 264 additional laborers to the NDS still requiring 1 million laborers over a reduced 61-year working timeframe. Therefore, while the level of National Security has not changed, 62 generations providing 16,129 laborers is approximately the same as 61 generations providing 16,393 laborers, and, as such, it has become more expensive. Two things to note: first, assuming that each living generation agrees to this new NDS, it will take 61 years to fully implement the new structure as each generation of 16,129 retires and expires, and a new generation of 16,393 laborers replaces it before structural harmony or equilibrium is once again achieved, and 79 years until all currently living generations have an equal TLC. Second, the percentage of society's population allocated to the NDS has changed from 1 million laborers in Y_{79}, to 1 million laborers and 16,129 retirees in Y_{80}, to 1 million laborers and 16,393 retirees in Y_{80+n} or any thereafter, which is an increase of 1.63%. If resources or laborers are confined to internal funding, the total compensation of actively working laborers would need to be reduced to allow for the additional retirement costs, or the 1 million laborers required for the NDS would have to be reduced by 16,393 to 983,607. If resources and or laborers are not confined to the current NDS system and can be funded externally, society must be able to absorb the loss of laborers supporting the NDS and the additional retirement costs. A one-year retirement structural benefit is relatively inexpensive as the years of service, or working years, are greater than the years of retirement, but if that relationship is reversed, there will be a compounding effect that will result in exponentially higher retirement costs for each generation after the generation that implements the structural change until harmony or equilibrium is once again achieved. Should the society and supporting population not have sufficient resources or production growth to keep up with this compounding effect in labor and retirement costs, it is very likely the system would either fail or some future generation will have to accept a lower, or at a minimum, different NDS structure while the unsustainable decisions of previous generations phase out over their lifetimes.

Overlapping Generations National Career Timeline, One Year of Retirement



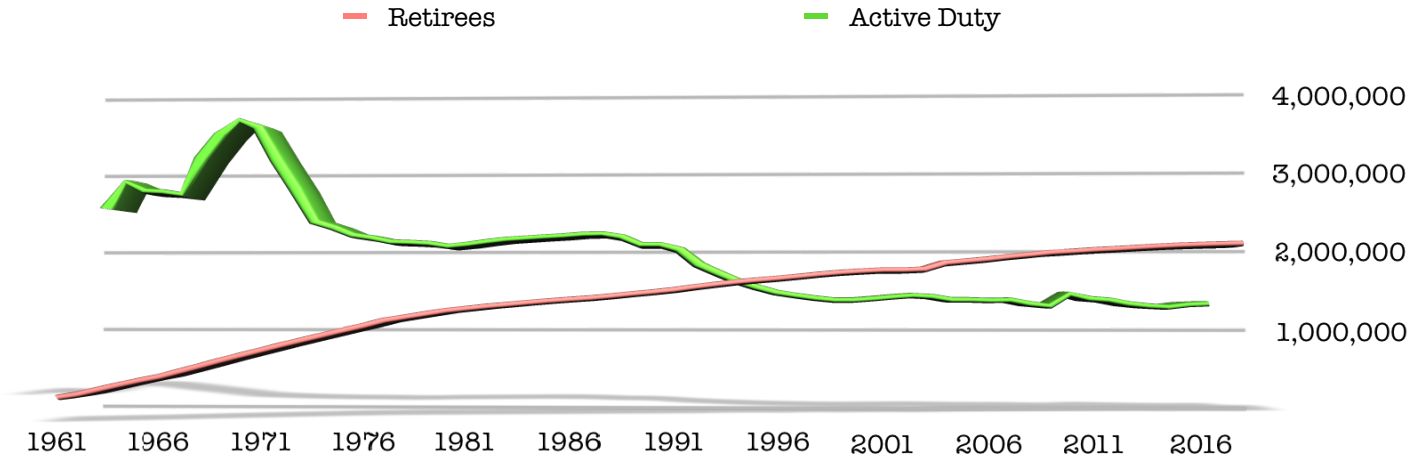
Another way to express the structural risk of retirement costs is to develop a table and increase retirement years and review the TLC once harmony or equilibrium is re-established to determine the impact on structural costs for all future generations. The following table reduces the average years of

service required by NDS laborers (green column) before allowing for retirement year benefits (red column) and assumes an accession age of 18 with a life expectancy of 80 years.

National Defense Structure Options					National Defense Structure Costs		
Years of Service	**Retired Year Benefit**	**Per Segment Laborers**	**Total Structured Laborers**	**Total Structured Retirees**	**Annual Total Laborer Costs**	**Annual Total Retirement Costs**	**Structured Total Labor Costs**
62	0	16,129	1,000,000	0	1,000,000*C	0*R	[79 * (1,000,000*C)] + [79 * (0*R)]
61	1	16,393	1,000,000	16,393	1,000,000*C	16,393*R	[79 * (1,000,000*C)] + [79 * (16,393*R)]
60	2	16,667	1,000,000	33,333	1,000,000*C	33,333*R	[79 * (1,000,000*C)] + [79 * (33,333*R)]
50	12	20,000	1,000,000	240,000	1,000,000*C	240,000*R	[79 * (1,000,000*C)] + [79 * (240,000*R)]
48	14	20,833	1,000,000	291,667	1,000,000*C	291,667*R	[79 * (1,000,000*C)] + [79 * (291,667*R)]
46	16	21,739	1,000,000	347,826	1,000,000*C	347,826*R	[79 * (1,000,000*C)] + [79 * (347,826*R)]
44	18	22,727	1,000,000	409,091	1,000,000*C	409,091*R	[79 * (1,000,000*C)] + [79 * (409,091*R)]
42	20	23,810	1,000,000	476,190	1,000,000*C	476,190*R	[79 * (1,000,000*C)] + [79 * (476,190*R)]
40	22	25,000	1,000,000	550,000	1,000,000*C	550,000*R	[79 * (1,000,000*C)] + [79 * (550,000*R)]
38	24	26,316	1,000,000	631,579	1,000,000*C	631,579*R	[79 * (1,000,000*C)] + [79 * (631,579*R)]
36	26	27,778	1,000,000	722,222	1,000,000*C	722,222*R	[79 * (1,000,000*C)] + [79 * (722,222*R)]
34	28	29,412	1,000,000	823,529	1,000,000*C	823,529*R	[79 * (1,000,000*C)] + [79 * (823,529*R)]
32	30	31,250	1,000,000	937,500	1,000,000*C	937,500*R	[79 * (1,000,000*C)] + [79 * (937,500*R)]
30	32	33,333	1,000,000	1,066,667	1,000,000*C	1,066,667*R	[79 * (1,000,000*C)] + [79 * (1,066,667*R)]
28	34	35,714	1,000,000	1,214,286	1,000,000*C	1,214,286*R	[79 * (1,000,000*C)] + [79 * (1,214,286*R)]
26	36	38,462	1,000,000	1,384,615	1,000,000*C	1,384,615*R	[79 * (1,000,000*C)] + [79 * (1,384,615*R)]
24	38	41,667	1,000,000	1,583,333	1,000,000*C	1,583,333*R	[79 * (1,000,000*C)] + [79 * (1,583,333*R)]
22	40	45,455	1,000,000	1,818,182	1,000,000*C	1,818,182*R	[79 * (1,000,000*C)] + [79 * (1,818,182*R)]
20	42	50,000	1,000,000	2,100,000	1,000,000*C	2,100,000*R	[79 * (1,000,000*C)] + [79 * (2,100,000*R)]
18	44	55,556	1,000,000	2,444,444	1,000,000*C	2,444,444*R	[79 * (1,000,000*C)] + [79 * (2,444,444*R)]
16	46	62,500	1,000,000	2,875,000	1,000,000*C	2,875,000*R	[79 * (1,000,000*C)] + [79 * (2,875,000*R)]
14	48	71,429	1,000,000	3,428,571	1,000,000*C	3,428,571*R	[79 * (1,000,000*C)] + [79 * (3,428,571*R)]
12	50	83,333	1,000,000	4,166,667	1,000,000*C	4,166,667*R	[79 * (1,000,000*C)] + [79 * (4,166,667*R)]
10	52	100,000	1,000,000	5,200,000	1,000,000*C	5,200,000*R	[79 * (1,000,000*C)] + [79 * (5,200,000*R)]

Allowing for one year of retirement benefits after 61 years of service increased NDS retirement costs by [79 * (16,393*R)]. If R is a reasonable retirement cost, it can be absorbed with little effect. Allowing for 12 years of retirement benefits increased NDS costs by [79 * (240,000*R)], a 1,364% increase in retirement costs over the one-year scenario. Allowing for 32 years of retirement after 30 years of service benefits increased NDS costs by [79 * (1,066,667*R)], a 6,407% increase in retirement costs over the one year scenario. Allowing for 42 years of retirement benefits after 20 years of service increased NDS retirement costs by [79 * (2,100,000*R)], a 12,710% increase in retirement costs over the one-year scenario; this scenario is similar to our current structural costs.

According to the 2018 Statistical Report on the Military Retirement System, 36,523 Active Duty and 12,232 Guard or Reserve members were added to the list of retirees receiving pay in 2018, bringing the total to 2,001,343. An increase in retirees for 58 straight years implies the structure is still attempting to find harmony or equilibrium. Additionally, the decline in Active Duty workers from a high of 3,547,902 in 1967 to 1,338,675 in 2018 also indicates an attempt to reduce current labor costs to adjust for predictable rising retirement costs, which totaled $58.57 billion in 2018 for pension pay- a number that does not include the substantial additional healthcare coverage sum provided at no-cost to retirees for life.



Source: https://actuary.defense.gov/Portals/15/Documents/MRS_StatRpt_2018%20v5.pdf?ver=2019-05-14-220629-787

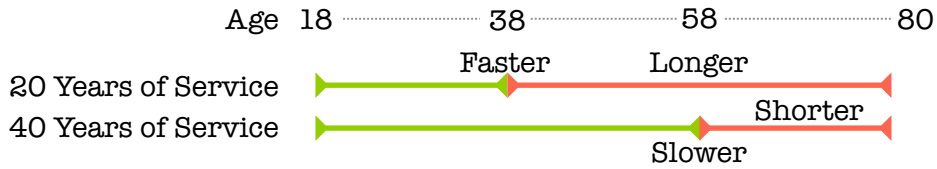
Reducing the average years of service of a laborer upon retirement causes a compounding effect in the anticipated structural retirement costs through three different factors.

= Working Years = Retired Years

Individuals Factors:

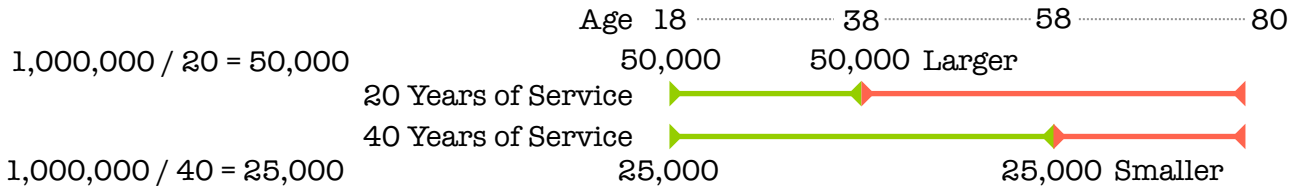
#1. Laborers enter the retirement pool faster (shorter years of service).

#2. Laborers remain in the retirement pool longer (longer retirement benefit period).

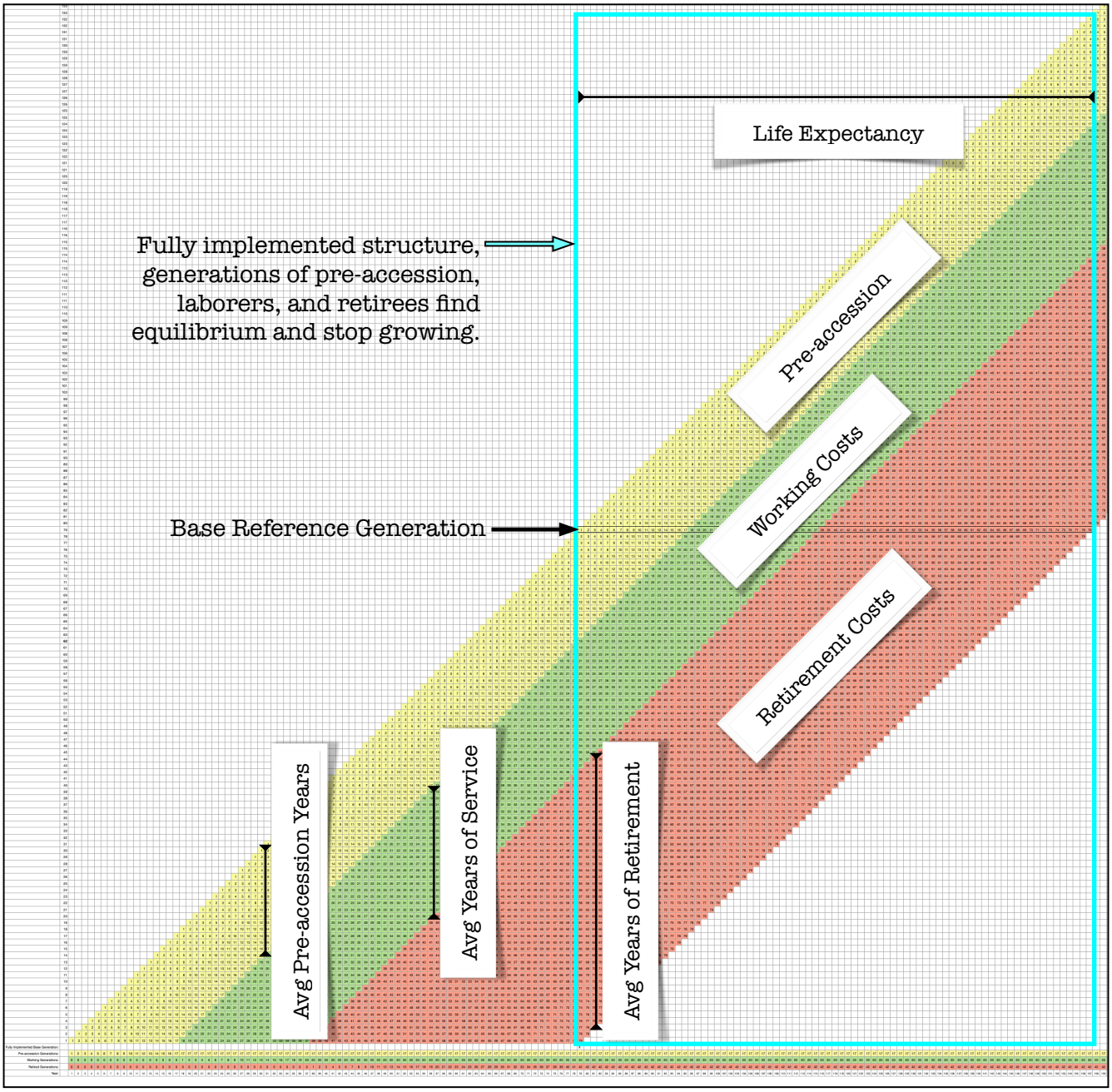


Structural Factor:

#3. Laborers enter the retirement pool in larger groups (larger replacement ratio).



Once a structure finds harmony or equilibrium, the number of generations in pre-accession, working, and retirement remains relatively constant. New generations replace passing generations, and the structure takes the form of a parallelogram that can be used to calculate the total structural labor costs. The pre-accession period incurs no costs but indirectly reduces retirement costs as it increases. The structural labor costs can be found taking the base (life expectancy) multiplied by height (average years of service) multiplied by segment size and working year compensation. Note, if there is a component of laborers that is not expected to add pension costs to the structure, the cost of this such component can be calculated by multiplying the number of its laborers by compensation with no need to account for retirement compensation. The structural retirement costs can be found by taking the base (life expectancy) multiplied by height (average years of retirement benefits) multiplied by segment size and retirement year compensation. The total structured costs divided by life expectancy provides an appropriate annual cost to compare different structural options.



I believe overlapping Generations of politically motivated Governments may have constructed a complex Unified Budget System to hide mathematical mistakes and unsustainable promises made to overlapping generations of citizens deliberately or accidentally. Government funds are being mis-labeled as “assets” when they are, in fact, liabilities, and associated interest is being mis-labeled as “interest income” when it is actually a compounding interest expense. Furthermore, budgetary costs are being mis-labeled as “interest expense" reducing transparency, and if the worst-case scenario is occurring, Government liabilities are being reduced by a discount factor that is actually working in the opposite direction of a normal discounting process resulting in a hidden compounding interest loop that will go exponential. Should my concerns be valid, as I know they are with an unwavering conviction that only mathematics can provide, Americans are going to receive a preventable lesson in the destructive capabilities of compounding debt. I am not afraid of being wrong, I am terrified of being right. If the Government’s Unified Budget is deliberately or accidentally hiding unsustainable structures embedded in the Office of Personnel Management Pension System, Military Pension System, Social Security Pension System, and Medicare Healthcare System, our darkest hour may still be before us.

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